

25X1

**Page Denied**

Next 1 Page(s) In Document Denied

US:



APPROACH  
THE MUTUAL ~~APPROACHMENT~~ OF AEROSOL PARTICLES IN A SOUND FIELD  
UNDER THE ACTION OF OSEEN'S HYDRODYNAMIC FORCES

[This is a translation of an article written by  
S. V. Pshenay-Severin in Doklady Akademii Nauk SSSR  
(Reports of the Academy of Sciences USSR), Vol 125,  
No 4, 1959, pages 775-778.]

In the studies of the mechanism of the acoustic coagulation of aerosols, hydrodynamic forces are usually regarded as one of the main causes of the ~~mutual approachment~~ <sup>approach</sup> of aerosol particles in a sound field. The effect of hydrodynamic forces depends on the velocity of the flow of the air medium around the particles,  $V$ . If this velocity of medium varies according to the law  $U = U_0 \sin \omega t$  ( $\omega = 2\pi f$ , where  $f$  is the frequency of sound), ~~then~~ the velocity of the steady-state motion of the individual particle equals  $v = U_0 n \sin(\omega t - \varphi)$  and the ~~resultant~~ <sup>moment-flow</sup> velocity  $V = U - v$  will be

$$V = U_0 m \cos(\omega t - \varphi). \quad (1)$$

$$\text{r.t. } m = \sin \varphi = \frac{\Omega}{(1+\Omega^2)^{1/2}}; \quad n = \cos \varphi = \frac{1}{(1+\Omega^2)^{1/2}}; \quad \Omega = \omega \tau; \quad \tau = \frac{2}{9} \frac{\rho}{\eta} R^2$$

where  $m = \sin \varphi = \frac{\Omega}{(1+\Omega^2)^{1/2}}; n = \cos \varphi = \frac{1}{(1+\Omega^2)^{1/2}}; \Omega = \omega \tau$ ;  
 $\tau = \frac{2}{9} \frac{\rho}{\eta} R^2$  is the relaxation time; and  $R$  is the particle radius  
( $\rho$  is density of particle matter,  $\eta$  is viscosity coefficient of the medium).

Hydrodynamic forces were investigated by Kirchhoff (Bibl. 1), Koenig (Bibl. 2), and ~~Bjerknes~~ Bjerknes (Bibl. 3) for the conditions of the flow of an ideal fluid around two spherical particles. If  $\psi_{12} = \pi/2$  (Fig. 1) then each of the two identical particles is acted upon by the force of attraction  $F \sim \rho R^6 v^2 / r_{12}^4$ , where  $\rho$  is density of medium. The question of the necessity of taking under consideration the effect of hydrodynamic forces on the process of acoustic coagulation was posed by Andrade (Bibl. 4). In the cases of interest to practice, the concentration of aerosols <sup>usually</sup> is relatively small and, consequently, the mean distance between particles  $\bar{r}_{12}$  is large. Inasmuch as the forces in

question decrease rapidly with increasing distances (as  $1/r_{12}^4$ ), therefore it is not possible to ~~adequately~~ explain satisfactorily by the action of these forces the process of the acoustic coagulation of aerosols (Bibl. 5) and, in particular, of natural fogs (Bibl. 6), for which  $R \sim 3 - 15$  microns. <sup>Thus</sup> ~~Moreover~~, the question of the role of hydrodynamic interaction in this process is not to be presumed as definitely clarified, on the basis of the following considerations. The above-named ~~researches~~ works were concerned solely with inertia forces in the ideal fluid, whereas, in the case of small aerosol particles, the corresponding Reynolds numbers are low ( $Re \sim 1 - 10$ ). Therefore, in addition to the inertia forces acting on the element of the medium it is also necessary to consider the viscosity forces.

Viscosity forces, together with inertia forces, can be computed on the basis of Oseen's hydrodynamic equation. When an individual spherical particle moves in <sup>an inert</sup> ~~viscous~~ fluid and in the case of an Oseenian mode of <sup>circumambient</sup> ~~streaming~~ flow at considerable distances  $r$  from the center of the sphericle, the streamlines in front of the sphericle and within a narrow area in the rear of the sphericle will be radial in nature and oriented in the same direction as the sphericle (cf. Bibl. 7, page 250). In this ~~in~~ connection, the magnitude of the speed of movement of the medium in rear of the sphericle decreases as ~~1/r~~, and in front of the sphericle -- as  $1/r^2$ . In the event ~~of~~ <sup>are</sup> two particles <sup>are</sup> situated comparatively close to each other, ~~each will disturb the ambient-flow field of the other~~ <sup>they will mutually</sup> ~~they will mutually~~ <sup>disturb the ambient-flow fields of each other.</sup> If the line of centers ~~of the particles~~ coincides with the direction of the ambient air flow (or diverges little from that direction), then, as a result of the interaction between the two particles, the resistance of each particle should diminish; <sup>however</sup> ~~as a consequence~~ of the difference in the magnitude of the speed of movement of the medium in front and in rear of each particle ~~the resistance~~, the diminution in resistance will be more considerable for the "head" particle than for the "tail" one. This difference in diminution of resistance is equivalent ~~to~~ to the effect of a force of attraction between the two

particles. The first to examine the hydrodynamic forces of this type was Oseen (Bibl. 8, 9).



~~XXXXXXXXXX~~

Fig. 1. Schematic Representation of the Positions of Interacting Particles.  $\psi_{12}$  -- angle between direction of ambient flow and the line of centers of the particles;  $r_{12}$  -- distance between centers of the particles.

Inasmuch as Oseen's hydrodynamic equation is a linear one, therefore when examining the interaction between the particles, it is possible to proceed from the assumption of the superposition of the fields of the flow around the particles and ~~XXXXXXXXXX~~ to estimate the magnitude of the resistance force acting on each particle, according to the formula

$$D_i^0 = 6\pi\eta R_i (V_i - u_k) S(Re_i),$$

where  $V_i = U - v_i$  is unperturbed ~~ambient-flow~~ velocity,  $u_k$  is perturbation of the ~~XXXXXXXXXX~~ velocity of the flow around the  $i$ -th particle, calculated according to the field of the flow around the  $k$ -th particle at the locus of the center of the  $i$ -th particle ( $i = 1, 2$ ;  $k = 2, 1$ ). In the given case the correction factor  $S$  is assumed equal to unity. If the particle with the subscript 1 is the "head" particle then, as a first approximation, when  $R_1 = R_2 = R$ ,  $\psi_{12} = 0$  and  $\varepsilon = R/r_{12} \ll 1$  the following expressions can be adopted for  $u_1$  and  $u_2$

$$u_1 = -\frac{3}{2} \varepsilon V_1, \quad u_2 = \frac{3\varepsilon^2}{Re} V_1. \quad (2)$$

In the event  $\varepsilon \ll 1$ , the magnitude of  $u_2$  is negligibly small compared with  $u_1$ , and it can be assumed that  $u_2 = 0$ ; here we arrive at the scheme of the unilateral hydrodynamic effect of the "head" particle on the "tail" particle, at which the computation of the relative movement of particles in a sound field is considerably simplified. The difference in resistance forces  $\Delta = D_1 - D_2$  is determined by perturbation  $u_1$ , the magnitude of which decreases with

increasing distance in inverse proportion only to the first power of  $r_{12}$  and not to the fourth power, as the magnitude of forces at potential ambient flow. Therefore, it can be expected that the effect of the interaction between particles on their relative movement will also be much greater at the Oseenian mode of <sup>ambient</sup> flow around them than at the potential one.

The afore-cited formulas are applicable, rigorously speaking, only at a stationary flow of the medium around the particles. In a sound field the velocity of the flow around particles varies in magnitude and in direction. However, if the frequency of sound is not high, it is apparently admissible to adopt the postulate of the quasistationarity of the processes occurring during the flow of air around particles with a radius of  $\sim 1 - 10$  microns (Bibl. 10). Inasmuch as at an Oseenian mode of ambient flow, the resistance force is always greater for the first particle than for the second, therefore in a sound field the ~~mutual displacement between~~ <sup>approach of</sup> particles should occur both when the <sup>ambient</sup> flow proceeds in one direction and when it is reversed. However, it is important to consider that the difference in the magnitude of the velocity of the medium in the front and in the rear of the sphericle and hence also the difference in the magnitude of resistance forces for the "head" and "tail" particles arises only at sufficiently high  $Re$ ; this difference disappears when  $Re \rightarrow 0$ .

Therefore, let us assume that in the sound field  $\Delta \neq 0$  only at those values of the <sup>ambient-flow</sup> ~~critical~~ velocity (1) at which the corresponding  $Re$  is greater than a certain ~~critical~~  $Re_{kp}$  (e. g.,  $Re_{kp} = 1$ ). Let  $t_1$  and  $t_2$  be the time instants ~~bounding~~ <sup>[vertical]</sup> bounding the interval within which  $|V| \geq v_{kp}$ . Let us calculate, in the case of  $\psi_{12} = 0$ , the elementary mutual displacement  $s_{12}$  of the identical particles relative to each other during <sup>the</sup> time interval  $(t_1, t_2)$ . The magnitude of  $s_{12}$  will characterize the result of the interaction between the particles during a half-period of sound vibrations  $T/2$ . Let us assume that prior to time instant  $t_1$  the velocity of flow around both particles changes according to law (1), while when  $t_1 \leq t \leq t_2$  the velocity of flow  $V_1$  around the particle with the subscript 1 (which is the "head" particle)

continues to vary according to formula (1) but the ~~the~~ velocity of flow around the second ("tail") particle is expressed in the form of  $\tilde{v}_2 = [(U - v_2) - \frac{1}{2}\epsilon v_1]$ . The equations of motion of the particles have the form of  $\tau \frac{dv_1}{dt} = V_1, \tau \frac{dv_2}{dt} = \tilde{v}_2$ . We consider the parameter  $\epsilon$  to be constant during the time  $\sim T/2$ .

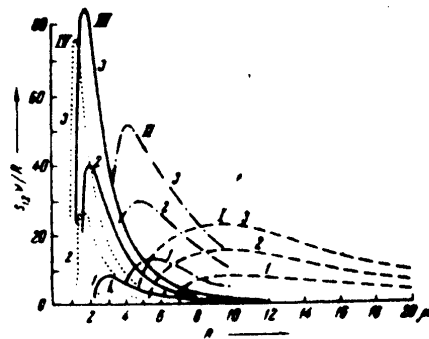
Let us introduce into consideration the magnitude  $v_{12} = v_1 - v_2$  and let us pass over to the dimensionless variables  $\theta = t/\tau$  and  $\xi_{12} = v_{12}/U_0$ ; then  $\xi_{12}$  is determined from the equation  $\frac{d\xi_{12}}{d\theta} + \xi_{12} = 1.5\epsilon m \cos(\Omega\theta - \varphi)$  on taking into account the initial condition  $\xi_{12}(\theta_1) = 0$ . Inasmuch as  $\frac{d}{d\theta}(\frac{v_{12}}{U_0}) = \xi_{12}$ , therefore the integration of the expression for  $\xi_{12}$  under the condition that  $s_{12}(\theta_1) = 0$ , makes it possible to obtain the displacement  $s_{12}$  during the time interval  $(t_1, t_2)$  in the form

$$\frac{s_{12}}{X_0} = -1.5\epsilon mn \{ [2mn \cos \Omega\theta_1 - (n^2 - m^2) \sin \Omega\theta_1] - \frac{m}{n} [2mn \sin \Omega\theta_1 + (n^2 - m^2) \cos \Omega\theta_1] e^{-(\theta_2 - \theta_1)} + (\sin \Omega\theta_2 - \frac{m}{n} \cos \Omega\theta_2) \}, \quad (3)$$

where  $X_0 = U_0/\omega$ .

Fig. 2 shows the results of the calculations of  $s_{12}$  for water droplets with various radii, on the basis of (3). The parameter  $\epsilon$  was assumed constant and equal to  $1 \cdot 10^{-2}$ ; this signifies that the droplets under study were always spaced apart from each other by a distance of  $r_{12} = 100 R$ , equal ~~to~~ approximately to  $\bar{r}_{12}$  when the concentration by weight of the droplets is one gram per cubic meter. The displacements  $s_{12}$  are related to the radius of particles  $R$ . To obtain ~~mutually~~ mutually comparable values at various sound frequencies  $f = \nu \cdot 10^2$  cycles per ~~second~~ second ( $\nu$  is an integer), the values of  $s_{12}/R$  are multiplied by the corresponding numbers  $\nu$ . The curves presented in Fig. 2 are distinguished by the presence of a maximum and also by their compression along the axis of the values of  $R$  and by the displacement of the maximum to the side of lower  $R$ s with increasing sound frequency (with increasing  $\nu$ ). At a given sound intensity (at a ~~given~~ given magnitude of  $U_0$ ) the magnitude of the maximum increases with increasing  $f$  only up to a limit, whereupon it begins to decrease. For a given  $R$  there exists an optimal value of sound frequency at which the maximal velocity of mutual <sup>u</sup> approach ~~of~~ of the droplets is present. Oseen's forces are effective for the droplets in the size

range of 1.5 to 15 microns. When  $R \sim 5-15$  microns, the most ~~xx~~ favorable sound frequencies are those of order several hundred cycles, and when  $R < 5$  microns, -- of order several ~~Q~~ kilocycles.



*Approach of*  
Fig. 2. Mutual ~~Rapprochement~~ Between Droplets of Various Radius According to Sound Frequency  $f = \nu$ ,  $10^2$  seconds $^{-1}$  and Sound Intensity (Velocity Amplitude  $U_0$ ). The curve group I corresponds to  $\nu = 1$ ; II --  $\nu = 5$ ; III --  $\nu = 50$ ; IV --  $\nu = 150$ . Curves 1 are computed for  $U_0 = 400$  cm/sec; 2 -- for  $U_0 = 700$  cm/sec; and 3 -- for  $U_0 = 1,000$  cm/sec.

The computed time  $s_{12}$  of the mutual ~~approach~~ <sup>approach</sup> of two particles from a distance equal to the mean distance between the particles to <sup>the zero contact</sup> distance at which they are in contact constitutes the characteristic time of the coagulation process, which can be compared with the experimentally observed time  $t_0$  of the 50-percent decrease in the count of particles (Bibl. 6). The observed time  $t_0$  for natural fogs is of ~~the~~ order of several minutes (Bibl. 6). If however we proceed from the values of  $s_{12}\nu/R$  cited in Fig. 2, then the time  $t_{12}$  will prove to be much shorter than one ~~second~~ second within a sufficiently broad range of sizes for each of the considered frequency values.\*

Thus, Oseen's hydrodynamic forces apparently are an important factor in the enlargement of droplets with radius of 1.5 to 15 microns in a sound field of a relatively low frequency, in addition to the process of

\*Inasmuch as the values of  $s_{12}$  are calculated on the basis of approximate

formulas which are not applicable for small distances, therefore, strictly speaking, these data can be used to judge about the rapprochement between particles from large distances  $r_{12} \sim 100 R$  to distances  $s_{12} \sim 10 R$  but <sup>to the contact distance</sup> ~~not to the contact distance~~.

orthokinetic coagulation. The afore-cited estimates are to be regarded as maximal ones requiring further refining and justification. It is necessary to <sup>investigate</sup> ~~do~~ more deeply into the conditions of the quasistationarity of the processes of the flow around particles, the interaction between particles at low velocities of ambient flow and at small mutual distances, and to compare the effect of hydrodynamic forces with the effect of orthokinetic coagulation. Most likely, the course of the over-all process is determined by the conjoint action of ~~XXXXX~~ ~~XXXXXX~~ both these factors.

The author wishes to express his deep appreciation to ~~XXXX~~ L. M. Levin for appraising the results of this study and for his counsel.

Institute of Applied Geophysics

Received

Academy of Sciences USSR

24 November 1958

#### Bibliography

1. Kirchhoff, G. "Mechanik," Leipzig, 1883
2. Koenig, W. "Ann. Phys. u. Chem.," 42, No 4, 549 (1891)
- ~~XXXXXX~~
3. Bjerkness [Bjerknes], V. "Vorlesungen ueber hydrodynamische Fernkraefte," Leipzig, 1900-1902
4. Da C. Andrade, E. N. "Trans. Farad. Soc.," 32, 1111 (1936)
5. Bradnt, O., Freund, H., and Hiedeman, E. "Koll. Zs.," 77, No 1, 103 (1936)
6. Sinclair, D. "Handbook on Aerosols," Washington, 1950
7. Slezkin, N. S. "Dynamics of Viscous Incompressible Fluids," Moscow, 1955
8. Oseen, C. W. "Ark. Mat., Astr. och Fysik," 7, No 33 (1912)
9. Oseen, C. ~~EX~~ W. "Hydrodynamik," Leipzig, 1927
10. Rytov, S. M., Vladimirovskiy, V. V., and Galanin, M. D. "ZhETF" [Journal of Experimental and Theoretical Physics], 8, 614 (1938)

END